**Homework #2**

**MSiA Program, Fall 2023**

**Optimization**

**Due: At the start of class on Monday 10/2/23**

1. Return to the Brewer’s Problem. Instead of just a capacity constraint on the three commodities, assume that each commodity has the following prices:

* Corn: $1.20 for the first 480 units, $2.00 for units 480 to 980, and $2.50 for any amount beyond that.
* Hops: $7.75 for the first 160 units, $8.25 for units 160 to 360. You do not have access to more than 360 for this buying period.
* Malt: $1.40 for the first 1190 units, $1.50 for units 1190 to 2190, and $1.55 for units 2190 to 3190. You do not have access to more than 3190 for this buying period.

Assume that the revenue for a unit of beer and ale is $100.

Formulate and solve this as a linear program. Your homework should include the following:

* 1. An explanation of the model
  2. The solution to the model
  3. An explanation of what the solution shows
  4. You should change some parameters so that the model can only use 480 of Corn, 160 of Hops, and 1190 of Malt. This will get it close to the original. You should run and confirm that the model makes the same decision as the original model with approximately the same profit.
  5. Go back to the model in part a,b, and c and run a scenario where the cost of corn goes down to $1.20 for any quantity. What is the solution and why did it pick this solution?
  6. Go back to the model in part a,b, and c and run a scenario where you have to make 3 more units of Ale than the current solution suggests. What is the answer? Comment on the result, why it happened, and what it means from a business point of view.

1. Cash Flow problem (You might need to spend time on this problem). See the Excel file *Pension Liability Student Data.* You are managing a pension fund and you need to make the payments shown at the end of each year for the next 14 years. For example, at the end of Year 1 you need to make a payment of 12 and at the end of Year 14 a payment of 31. To cover these payments, you can buy as many of the three bonds shown on the spreadsheet (and you can buy a fraction of a bond). You will purchase the bonds at the beginning of Year 1. Each year of the bond you get the coupon value as interest. In the year of maturity, you get the coupon value plus the value at maturity. For example, Bond 1 will cost you .98. At the end of Years 1-4 you get .06. At the end of year 5, you get 1.06 (coupon + value). In addition, you can invest in savings that pay an interest rate of 4%. Assume that you collect the coupons, value of the bond, and savings interest rates at the end of the year, and then you pay the pension liability for that year immediately after that. Set this problem up as a linear program and solve it (figure out what the objective is). Write a brief explanation of the answer. Here are some hints
   1. Think about this model as a flow model. In any given year, the amount flowing into that year must equal the amount flowing out.
      1. So, at the start of Year 1, you make an initial investment (flow in) in four things- the 3 Bonds and a Savings account (these are the flow out).
      2. At the end of Year X, the flow in would be your coupons and any principal from the bonds (paid out in years 5, 11, and 14), your principal and interest from savings, and your flow out would be the pension payment and your reinvestment back in savings (you only get to buy the bonds at the start).
   2. You will need to figure out the objective and the decision variables. The decision variables may not be so obvious.

1. Return to the “Two-Products, Three-Machine” problem from Homework #1. Assume that the sales team has landed more contracts and now the minimum market requirement is 100 for both products. The maximum is still the same. What happens when you run and why? Now (and this is the important part of this question), assume that you want to give this model to the VP of Sales and the VP will want to try out many different market minimums and maximums. Set up the model so that it always runs and provides the VP with valuable information. Note that we are not changing the underlying constraints—so the plant stays the same. But, your model needs to run. Assume that the VP is not all that technically savvy and will not hesitate to call you with urgent tech support calls—and you don’t want to be bugged with urgent tech support calls. (This is a problem about good ***model*** design, but not look & feel—don’t work on a GUI). (I want you to think about this problem)